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GAUGE DEPENDENCE OF ULTRAVIOLET BEHAVIOUR OF QCD

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As is well known, the two-loop contribution to the beta-function of the QCD running coupling $\bar{\alpha}_s$ can depend on the gauge parameter a. In this paper the results of renormalization-group (RG) analysis of several MOM schemes with this dependence are presented. It is shown that for some cases gauge dependence can essentially influence the ultraviolet behaviour of $\bar{\alpha}_s$ and, particularly, destroy the asymptotic freedom property. Possible "physical" implications of these phenomena are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Калибровочная зависимость ультрафиолетового поведения в КХД

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Как известно, двухпетлевой коэффициент бета-функции эффективного заряда КХД $\bar{\alpha}_s$ может содержать зависимость от калибровочного параметра. В работе выполнен ренормгрупповой анализ нескольких МОМ-схем, в которых такая зависимость имеет место. Показано, что в ряде случаев калибровочная зависимость существенно влияет на ультрафиолетовое поведение $\bar{\alpha}_s$ и, в частности, может приводить к нарушению асимптотической свободы. Обсуждены возможные следствия этого феномена для физических величин.

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1. INTRODUCTION

Experiments that will be accessible in the near future can provide a possibility of more accurate comparison of QCD predictions. We have in mind the check of multi-loop contributions as well as effects of heavy quark masses. This especially concerns the 2-loop corrections.

The results of 2-loop calculations depend on the renormalisation scheme as well as on the choice of gauge. The scheme and gauge dependence of 2-loop QCD approximation has been considered in papers /1-5/. It was found /1/ that in MOM schemes 2-loop contribution to the beta-function of the QCD effective coupling $\bar{\alpha}_s$ (α_s being equal to $g^2/4\pi$) does depend on the gauge parameter a. Due to this, the ultraviolet asymptotic (UVA) behaviour of $\bar{\alpha}_s$ must be defined by the RG analysis of the system of two differential equations

$$\frac{d\bar{\alpha}(\ell)}{d\ell} = \beta(\bar{\alpha}, \bar{a}), \quad \frac{d\bar{a}(\ell)}{d\ell} = \bar{a}\, b(\bar{\alpha}, \bar{a}), \quad \ell = \ln\frac{Q^2}{\mu^2} \tag{1}$$

with boundary conditions: $\bar{\alpha}(0) = \alpha$, $\bar{a}(0) = a$ (where the subscript "s" in α_s is omitted). Here it is essential that the group generator β depends on the running gauge parameter $\bar{a}$.

In this paper we present the results of detailed analysis of the system (1) taken in 2-loop approximation for five different MOM schemes which are more often used in calculation of observed processes. It is shown that the UVA behaviour can essentially depend on the gauge parameter initial value $a(0) = a$. In three of our schemes for negative a, the $\bar{\alpha}$ UVA is governed by a fixed point in the phase plane $(\bar{a}, \bar{\alpha})$, see fig. 1. The coordinates of this point depend upon a renormalization scheme and flavour number f - see the Table. In all five schemes in some part of the phase plane for $a > a_*(\alpha) > 0$ solutions for $\bar{\alpha}(\ell)$ exist only in the finite interval of logarithmic argument $\ell < \ell^*$ as at $\ell = \ell^*$ the running $\bar{a}$ has a pole (the trouble of "zero-change" type). For the cases when the asymptotic freedom (AF) for $\bar{\alpha}$ takes place we give relation between the scale parameter Λ for minimal subtraction scheme and its value for momentum subtraction schemes. This relation differs from the analogous one presented in paper /2/.

2. FORMALISM

We limit ourselves to the case when the gauge is fixed in a covariant way by the term $= -(\partial_\mu A^\mu)^2/2a$ in the Lagrangian. Hence $a = 0$ corresponds to transverse (Landau) gauge and $a = 1$ to the diagonal (Feynman) one. Here, the RG solutions for Green functions and matrix elements can be expressed though the effective (i.e. running) coupling $\bar{\alpha}$ and gauge $\bar{a}$, which can be found from system (1).

The generators β and b in perturbation theory can be expressed in the form

$$\begin{aligned} \beta(\alpha, a) &= -\beta_1 \frac{\alpha^2}{4\pi} - \beta_2(a)\frac{\alpha^3}{(4\pi)^2} - \ldots, \\ b(\alpha, a) &= b_1(a)\frac{\alpha}{4\pi} + b_2(a)\left(\frac{\alpha}{4\pi}\right)^2 + \ldots . \end{aligned} \tag{2}$$

To find 2-loop terms for MOM schemes it is possible to start with β_2 and b_2 for $\overline{MS}$ and make recalculation using relations between renormalized α, a in a given MOM scheme and those in $\overline{MS}$ scheme

$$\alpha = \alpha_{\overline{MS}} + Q(a_{\overline{MS}})\,\alpha^2_{\overline{MS}}/4\pi + \dots ,$$
$$a = a_{\overline{MS}}(1 + K(a_{\overline{MS}})\,\alpha_{\overline{MS}}/4\pi + \dots). \qquad (3)$$

Then, wanted β_2 and b_2 are expressed through $\beta_2^{\overline{MS}}$ and $b_2^{\overline{MS}}$ with the help of Q and K which can be found from 1-loop calculations. The corresponding relations are

$$\beta_1 = \beta_1^{\overline{MS}} = 11 - \frac{2}{3} f, \quad b_1 = b_1^{\overline{MS}}(a) = \frac{13-3a}{2} - \frac{2}{3} f,$$
$$\beta_2(a) = \beta_2^{\overline{MS}} - b_1(a)\, a \frac{\partial Q(a)}{\partial a}, \qquad (4)$$
$$b_2(a) = b_2^{\overline{MS}}(a) + b_1(a)\left[-Q(a) + a\frac{\partial K(a)}{\partial a}\right] - K(a)\left[\beta_1 + a\frac{\partial b_1(a)}{\partial a}\right].$$

Here[5]

$$\beta_2^{\overline{MS}} = 102 - \frac{38}{3} f,$$
$$b_2^{\overline{MS}}(a) = \frac{531 - 99a - 18a^2}{8} - \frac{61}{6} f, \qquad (5)$$
$$K = -\frac{97}{12} - \frac{3}{2} a - \frac{3}{4} a^2 + \frac{10}{9} f.$$

In contradistinction to K that is universal, as it is related only to 1-loop gluon propagator renormalization, the coefficient Q depends on the choice of a particular MOM scheme[2,4,6]

I) $$Q = \frac{169}{12} + \frac{9a}{2} + \frac{3a^2}{4} - \frac{10}{9} f \quad \text{for} \quad \alpha = \Gamma_{A\bar{\eta}\eta}(-\mu^2, -\mu^2, 0)$$

II) $$Q = \frac{223}{12} + 3a + \frac{3a^2}{4} - \frac{10}{9} f \quad \text{for} \quad \alpha = \Gamma_{A\bar{\eta}\eta}(0, -\mu^2, -\mu^2)$$

III) $$Q = \frac{205}{12} + \frac{9}{2} a + \frac{3}{4} a^2 + R\,\frac{5 - 8a - a^2}{8} - \frac{10}{9} f, \qquad (6)$$

$$\text{for} \quad \alpha = \Gamma_{A\bar{\eta}\eta}(-\mu^2, -\mu^2, -\mu^2)$$

(IV) $Q = 22 + \frac{9}{4} a - \frac{3a^2}{2} + \frac{a^3}{4} + R \frac{23 - 27a + 6a^2}{12} - f \frac{12 + 8R}{9}$,

for $\alpha = \Gamma_{AAA}(-\mu^2, -\mu^2, -\mu^2)$

(V) $Q = \frac{41}{2} - \frac{23}{6} a - R \frac{35 + a}{9} - \frac{10}{9} f$, for $\alpha = \Gamma_{A\bar{q}q}(-\mu^2, -\mu^2, -\mu^2)$ (6)

Here

$$R = - \int_0^1 \frac{2 \ln x \, dx}{1 - x + x^2} = 2.3439072\ldots$$

and subscripts A, η, q for vertices Γ correspond to the gluon, Faddeev-Popov ghost and quark fields.

3. RESULTS

We present a brief review of our numerical analysis demonstrating a wide scope of diverse type of UVA behaviour possible in different MOM schemes. First, three of our schemes obey the fixed point in the left quadrant, i.e. for some $\bar{\alpha} = \alpha_\infty$, $a = a_\infty < 0$, where both the generators β and b are equal to zero. The coordinates α_∞ and a_∞ in these schemes for several values of flavour number are given in the Table.

Table

Scheme	f	α_∞	a_∞
I	3	1.36760	−4.7632
	4	1.35767	−4.6838
	5	1.33731	−4.5961
	6	1.30338	−4.4983
II	3	0.51311	−5.0798
	4	0.51997	−4.9866
	5	0.52567	−4.8856
	6	0.52956	−4.7754
III	3	0.26327	−7.2283
	4	0.27860	−7.0077
	5	0.29556	−6.7757
	6	0.31413	−6.5296

The behaviour of integral curves for the scheme III are qualitatively shown in the left half of Fig.1. For scheme I the left singular point is a stable knot. In its vicinity $\bar{a}(\ell) = a_\infty + A\exp(-\rho\ell)$, $\rho > 0$. However, in the schemes II and III this point turns out to be a focus around which

$$\bar{a}(\ell) = a_\infty + A\exp(-\rho\ell) \cdot \sin[\sigma\ell + r].$$

It must be noted that from the analysis of the behaviour of the curves $a_\beta(a)$, $a_b(a)$ defined by the relations $\beta(a_\beta, a) = 0$, $b(a_b, a) = 0$ and our estimate of the magnitude of 3-loop correction to β it is possible to conclude that the effects of 3-loop contribution on the fixed point position (and, possibly, on existence) should be essential.

All five schemes have a stable singular point at $a = a^* = (39-4f)/9$, to which tend solutions from a part of the right quadrant. Solutions from the other part have a quite different UVA behaviour ($\bar{a} \to \infty, \bar{\alpha} \to 0$ or, in scheme V, to finite value) of "ghost-trouble" type. The scheme IV has only these singularities. In the scheme V the type of the UVA behaviour of $\bar{\alpha}(\ell)$ is essentially dependent on quark number f as well as on quadrant of phase plane: at $a < 0$ there exists a singular point for $f \geq 5$ and in $a > 0$ quadrant — second singular point for $f \geq 4$ values. At the same time for $f = 3$ there exists a region $a > a_0 > 0$ in which our system (1) obeys the solution only for finite values of logarithmic arguments $\ell < \ell_*$. At $\ell = \ell_*$ the effective coupling $\bar{\alpha}(\ell)$ is finite but the effective gauge $\bar{a}$ has a pole singularity (of "zero of the charge" type). The behaviour of the phase curve for this case is qualitatively presented by the dotted line in the upper right part of Fig.1.

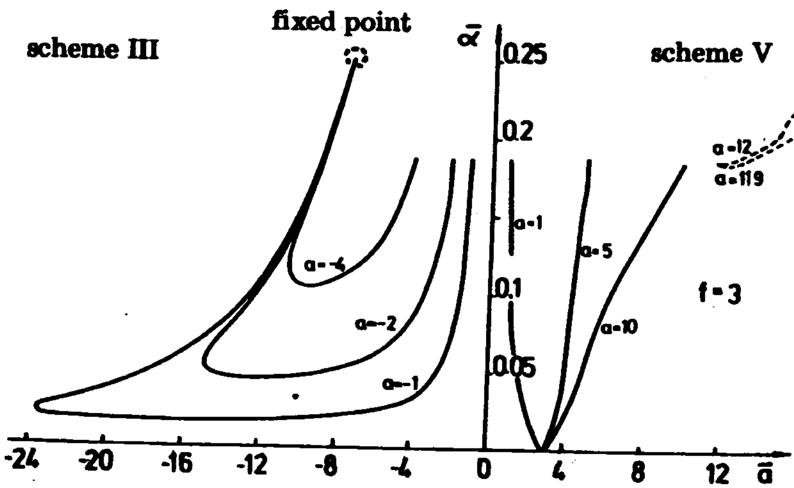

Fig.1

In Fig.2 we show the $\bar{a}(\ell)$ dependence for several cases with a ≤ 0 for scheme III. For all our calculations we started from the effective coupling boundary value $\bar{a}(0)=0.19$.

In Fig.3 we give the $\bar{a}(\ell)$ dependnece for scheme V for several positive a values.

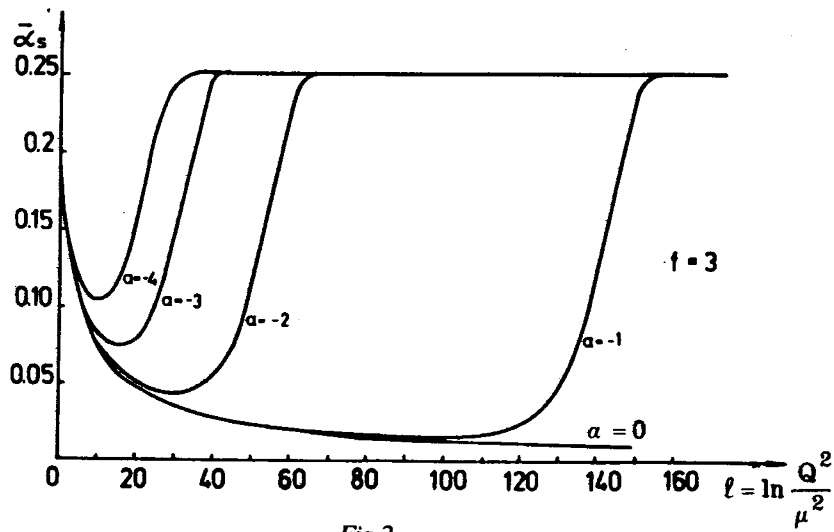

Fig.2

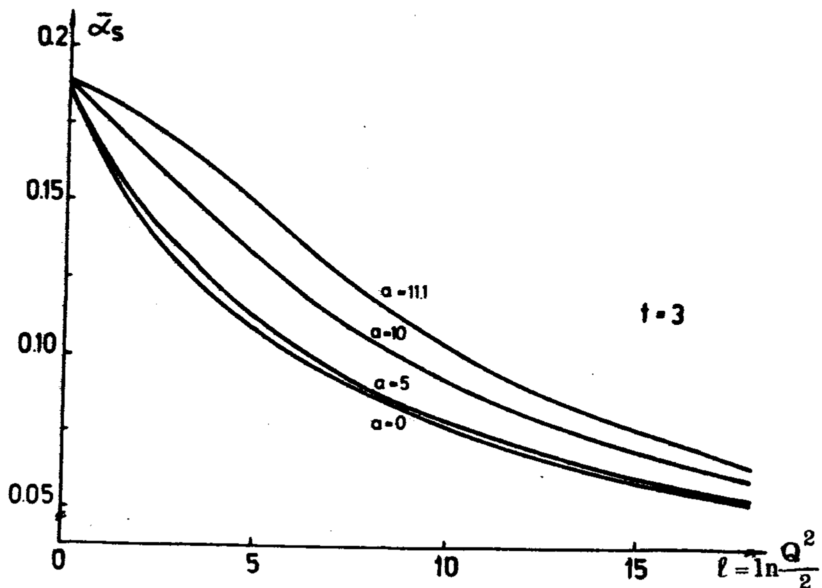

Fig.3

Note here that for the cases when the AF property is not destroyed, there exists a simple relation between the scale parameters Λ_{MOM} and $\Lambda_{\overline{MS}}$:

$$\Lambda_{MOM}(a) = \Lambda_{\overline{MS}} \exp\{Q(a)/2\beta_1\} \tag{7}$$

which was obtained in paper [2] where the gauge parameter a was considered as a fixed one. With due account that it "runs" it is necessary to modify this relation by substituting instead of a its limiting UVA value

$$\Lambda_{MOM}(a) \to \Lambda_{MOM}(a^*) \tag{8}$$

as it follows from the integral curve behaviour presented in the right quadrant of Fig.1. Here, from a practical point of view the equation (7) can be considered as acceptable for a rather large interval of variable ℓ. However, strictly speaking Λ varies with $\bar{a}$ (i.e. with ℓ) and for the description of "real asymptotics", — e.g. in GUT region — it is necessary to use the limiting values as expressed by equation (8).

It is worth mentioning that more accurate analysis needs the inclusion of heavy quark masses that can be performed on the basis of the corresponding RG formalism [7]. It is essential that in this generalization the gauge dependence can be important on the one-loop level.

Note added in proof: When our calculations have been completed we received paper [9] in which part of results for the case I are obtained. In this connection we must mention that qualitatively the effect of existing stable point and destroying the AF property in left quadrant for the schemes I, II was discovered by V.V.Vladimirov and one of us (O.T.) in 1982.

4. PHYSICAL DISCUSSION

The results obtained can be considered as paradoxical, as "QCD practitioners" usually treat the effective coupling $\bar{a}$ as an observable object: it is "measured" by experiment; the physical content of asymptotic freedom phenomena is expressed with the help of $\bar{a}$ behaviour, the UV extrapolation of $\bar{a}$ (as well as $\bar{a}_1$ and $\bar{a}_2$ of electroweak theory) forms the basis of numerical estimations in the GUT speculations (leptoquark masses and proton decay rate).

Our analysis reveals that, from the principal point of view, the question of gauge dependence merits as much attention as the problem of scheme dependence. From general arguments it follows that the observed quantities (transition matrix elements) as a whole must be gauge independent. This property can be formulated for each given order of perturbation thery (as it is well known from the practice of QED calculations). However, theoretical results referred to the concrete renormalization scheme, include effects of the RG summation of infinite sequences of the leading and subleading logarithms in all orders of perturbation theory. Bisides, in the RG calculations we start with approximate expression obtained by truncating the expansion in noncovariant object. Due to this, the property of gauge invariance, like the scheme independence, can be easily violated. Its restoration needs a special procedure.

From the practical point of view, it is safely enough to use more popular values $a = 0$ and $a = 1$ because, as follows from our analysis, in the strip $0 \leq a \leq a^*$ the effective coupling with sufficient accuracy can be considered as independent of a. Nevertheless, in some situations, for example, in infrared QCD analysis (confinement problem), it may happen to be more preferable to use gauge parameter value out of the safety region. Thus for the Arbuzov gauge ($a = -3$)[8] it is necessary to be careful in conjunction of the results of infrared analysis with the UVA.

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